EXERCISE–5

Part : (A) Only one correct option

1. $\int [f(x)g''(x) - f''(x)g(x)]\, dx$ is equal to

(A) $\dfrac{f(x)}{g'(x)}$ (B) $f'(x)\, g(x) - f(x)\, g'(x)$
(C) $f(x)\, g'(x) - f'(x)\, g(x)$ (D) $f(x)\, g'(x) + f'(x)\, g'(x)$

2. $\int \dfrac{1}{\sqrt{\sin^3 x \cos x}}\, dx$ is equal to

(A) $\dfrac{-2}{\sqrt{\tan x}} + c$ (B) $2\sqrt{\tan x} + c$ (C) $\dfrac{2}{\sqrt{\tan x}} + c$ (D) $-2\sqrt{\tan x} - c$

3. $\int \dfrac{\ell n\,|x|}{x\sqrt{1 + \ell n\,|x|}}\, dx$ equals :

(A) $\dfrac{2}{3}\sqrt{1 + \ell n\,|x|}\,(\ell n\,|x| - 2) + c$ (B) $\dfrac{2}{3}\sqrt{1 + \ell n\,|x|}\,(\ell n\,|x| + 2) + c$
(C) $\dfrac{1}{3}\sqrt{1 + \ell n\,|x|}\,(\ell n\,|x| - 2) + c$ (D) $2\sqrt{1 + \ell n\,|x|}\,(3\,\ell n\,|x| - 2) + c$

4. If $\int \dfrac{x \tan^{-1} x}{\sqrt{1+x^2}}\, dx = \sqrt{1+x^2}\, f(x) + A\ \ell n\,(x + \sqrt{x^2+1}) + C$, then

(A) $f(x) = \tan^{-1} x$, $A = -1$ (B) $f(x) = \tan^{-1} x$, $A = 1$
(C) $f(x) = 2\tan^{-1} x$, $A = -1$ (D) $f(x) = 2\tan^{-1} x$, $A = 1$

5. $\int \dfrac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\, dx =$

(A) $\dfrac{1}{2}\sin 2x + c$ (B) $-\dfrac{1}{2}\sin 2x + c$ (C) $-\dfrac{1}{2}\sin x + c$ (D) $-\sin^2 x + c$

6. $\int \sqrt{\dfrac{a+x}{a-x}} - \sqrt{\dfrac{a-x}{a+x}}\, dx$ is equal to

(A) $-2\sqrt{a^2 - x^2} + C$ (B) $\sqrt{a^2 - x^2} + C$ (C) $-\sqrt{x^2 - a^2} + C$ (D) none of these

7. $\int \tan(x-\alpha)\tan(x+\alpha)\tan 2x\, dx$ is equal to

(A) $\ell n \left|\dfrac{\sqrt{\sec 2x}\,.\,\sec(x+\alpha)}{\sec(x-\alpha)}\right| + C$ (B) $\ell n \left|\dfrac{\sqrt{\sec 2x}}{\sec(x-\alpha)\sec(x+\alpha)}\right| + C$

(C) $\ell n \left|\dfrac{\sqrt{\sec 2x}\,.\,\sec(x+\alpha)}{\sec(x+\alpha)}\right| + C$ (D) none of these

8. $\int \sqrt{\sec x - 1}\, dx$ is equal to

(A) $2\,\ell n\left(\cos\dfrac{x}{2} + \sqrt{\cos^2\dfrac{x}{2} - \dfrac{1}{2}}\right) + C$ (B) $\ell n\left(\cos\dfrac{x}{2} + \sqrt{\cos^2\dfrac{x}{2} - \dfrac{1}{2}}\right) + C$

(C) $-2\,\ell n\left(\cos\dfrac{x}{2} + \sqrt{\cos^2\dfrac{x}{2} - \dfrac{1}{2}}\right) + C$ (D) none of these

9. $\int \dfrac{dx}{\cos^3 x\sqrt{\sin 2x}}$ is equal to

(A) $\sqrt{2}\left(\sqrt{\cos x} + \dfrac{1}{5}\tan^{5/2} x\right) + C$ (B) $\sqrt{2}\left(\sqrt{\tan x} + \dfrac{1}{5}\tan^{5/2} x\right) + C$

(C) $\sqrt{2}\left(\sqrt{\tan x} - \dfrac{1}{5}\tan^{5/2} x\right) + C$ (D) none of these

10. Primitive of $\dfrac{3x^4 - 1}{(x^4 + x + 1)^2}$ w.r.t. x is:

(A) $\dfrac{x}{x^4 + x + 1} + c$ (B) $-\dfrac{x}{x^4 + x + 1} + c$

(C) $\dfrac{x + 1}{x^4 + x + 1} + c$ (D) $-\dfrac{x + 1}{x^4 + x + 1} + c$

11. If $\int \dfrac{x^4 + 1}{x(x^2 + 1)^2}\,dx = A\,\ell n\,|x| + \dfrac{B}{1 + x^2} + c$, where c is the constant of integration then:

(A) A = 1; B = – 1 (B) A = – 1; B = 1
(C) A = 1; B = 1 (D) A = – 1; B = – 1

12. $\int \sqrt{\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}}\,dx$ equals :

(A) $\sqrt{x}\,\sqrt{1 - x} - 2\sqrt{1 - x} + \cos^{-1}(\sqrt{x}) + c$ (B) $\sqrt{x}\,\sqrt{1 - x} + 2\sqrt{1 - x} + \cos^{-1}(\sqrt{x}) + c$

(C) $\sqrt{x}\,\sqrt{1 - x} - 2\sqrt{1 - x} - \cos^{-1}(\sqrt{x}) + c$ (D) $\sqrt{x}\,\sqrt{1 - x} + 2\sqrt{1 - x} - \cos^{-1}(\sqrt{x}) + c$

13. $\int$ sin x. cos x. cos 2x. cos 4x. cos 8x. cos 16 x dx equals:

(A) $\dfrac{\sin 16x}{1024} + c$ (B) $-\dfrac{\cos 32x}{1024} + c$ (C) $\dfrac{\cos 32x}{1096} + c$ (D) $-\dfrac{\cos 32x}{1096} + c$

14. $\int \dfrac{1}{\cos^6 x + \sin^6 x}\,dx$ equals :

(A) $\tan^{-1}(\tan x + \cot x) + c$ (B) $-\tan^{-1}(\tan x + \cot x) + c$
(C) $\tan^{-1}(\tan x - \cot x) + c$ (D) $-\tan^{-1}(\tan x - \cot x) + c$

15. $\int\left\{\ln(1+\sin x) + x\tan\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)\right\}dx$ is equal to:

(A) $x\,\ell n\,(1 + \sin x) + c$ (B) $\ell n\,(1 + \sin x) + c$ (C) $-x\,\ell n\,(1 + \sin x) + c$ (D) $\ell n\,(1 - \sin) + c$

16. $\int \dfrac{dx}{\cos^3 x\,.\,\sqrt{\sin 2x}}$ equals:

(A) $\dfrac{\sqrt{2}}{5}(\tan x)^{5/2} + 2\sqrt{\tan x} + c$ (B) $\dfrac{\sqrt{2}}{5}(\tan^2 x + 5)\sqrt{\tan x} + c$

(C) $\dfrac{\sqrt{2}}{5}(\tan^2 x + 5)\sqrt{2\tan x} + c$ (D) none

17. If $\int \dfrac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + c$ where c is an arbitrary constant of integration then the values of 'a' and 'b' are respectively:

(A) $-2\ \&\ \dfrac{2}{3}$ (B) $2\ \&\ -\dfrac{2}{3}$ (C) $2\ \&\ \dfrac{2}{3}$ (D) none

18. $\int \dfrac{x^2 - 1}{x^3\sqrt{2x^4 - 2x^2 + 1}}\,dx$ is equal to **[IIT - 2006, (3, –1)]**

(A) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + c$ (B) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + c$

(C) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x} + c$ (D) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + c$

Part : (B) May have more than one options correct

19. If $\int \dfrac{(x - 1)\,dx}{x^2\sqrt{2x^2 - 2x + 1}}$ is equal to $\dfrac{\sqrt{f(x)}}{g(x)} + c$ then

(A) $f(x) = 2x^2 - 2x + 1$ (B) $g(x) = x + 1$

(C) $g(x) = x$ (D) $f(x) = \sqrt{2x^2 - 2x}$

20. $\int \frac{dx}{5 + 4\cos x} = I \tan^{-1}\left(m \tan\frac{x}{2}\right) + C$ then:

(A) $I = 2/3$ (B) $m = 1/3$ (C) $I = 1/3$ (D) $m = 2/3$

21. If $\int \frac{3\cot 3x - \cot x}{\tan x - 3\tan 3x}\, dx = p\, f(x) + q\, g(x) + c$ where 'c' is a constant of integration, then

(A) $p = 1;\ q = \frac{1}{\sqrt{3}};\ f(x) = x;\ g(x) = \ell n \left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right|$

(B) $p = 1;\ q = -\frac{1}{\sqrt{3}};\ f(x) = x;\ g(x) = \ell n \left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right|$

(C) $p = 1;\ q = -\frac{2}{\sqrt{3}};\ f(x) = x;\ g(x) = \ell n \left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right|$

(D) $p = 1;\ q = -\frac{1}{\sqrt{3}};\ f(x) = x;\ g(x) = \ell n \left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right|$

22. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx$ is equal to:

(A) $\cot^{-1}(\cot^2 x) + c$ (B) $-\cot^{-1}(\tan^2 x) + c$
(C) $\tan^{-1}(\tan^2 x) + c$ (D) $-\tan^{-1}(\cos 2x) + c$

23. $\int \frac{\ell n\left(\frac{x-1}{x+1}\right)}{x^2 - 1}\, dx$ equal:

(A) $\frac{1}{2}\ \ell n^2 \frac{x-1}{x+1} + c$ (B) $\frac{1}{4}\ \ell n^2 \frac{x-1}{x+1} + c$ (C) $\frac{1}{2}\ \ell n^2 \frac{x+1}{x-1} + c$ (D) $\frac{1}{4}\ \ell n^2 \frac{x+1}{x-1} + c$

24. $\int \frac{\ell n(\tan x)}{\sin x\ \cos x}\, dx$ equal:

(A) $\frac{1}{2}\ \ell n^2(\cot x) + c$ (B) $\frac{1}{2}\ \ell n^2(\sec x) + c$

(C) $\frac{1}{2}\ \ell n^2(\sin x \sec x) + c$ (D) $\frac{1}{2}\ \ell n^2(\cos x\ \mathrm{cosec}\, x) + c$

EXERCISE–6

1. Integrate with respect $\int \frac{\mathrm{cosec}^2 x . \sin x}{(\sin x - \cos x)} . dx$

2. Integrate with respect to x $\frac{1 - x^2}{1 - x^2 + x^4}$

3. Integrate with respect to x $\frac{1}{(x+1)\sqrt{x^2 + 2}}$

4. $\int \frac{(x-1)^2}{x^4 + x^2 + 1}\, dx$

5. $\int \frac{2\sin 2\phi - \cos\phi}{6 - \cos^2\phi - 4\sin\phi}\, d\phi$

6. $\int \frac{\tan\theta + \tan^3\theta}{1 + \tan^3\theta}\, d\theta$

7. $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x}\, dx$

8. $\displaystyle\int \frac{3+4\sin x+2\cos x}{3+2\sin x+\cos x}\,dx$

9. $\displaystyle\int \frac{1+\cos\alpha\ \cos x}{\cos\alpha+\cos x}\,dx$

10. $\displaystyle\int \frac{dx}{(x-\alpha)\sqrt{(x-\alpha)(x-\beta)}}$

11. $\displaystyle\int \frac{dx}{\left(x^3+3x^2+3x+1\right)\sqrt{x^2+2x-3}}$

12. $\displaystyle\int e^x\frac{x^3-x+2}{\left(x^2+1\right)^2}\,dx$

13. $\displaystyle\int \frac{(\cos 2x-3)}{\cos^4 x\sqrt{4-\cot^2 x}}\,dx$

14. $\displaystyle\int \left[\frac{\sqrt{x^2+1}\,\left\{\ln\left(x^2+1\right)-2\ln x\right\}}{x^4}\right]dx$

15. $\displaystyle\int \frac{dx}{(a+b\cos x)^2}$, $(a > b)$

16. $\displaystyle\int \sqrt{\frac{\operatorname{cosec} x-\cot x}{\operatorname{cosec} x+\cot x}}\cdot\frac{\sec x}{\sqrt{1+2\sec x}}\,dx$

17. $\displaystyle\int \frac{x}{\left(7x-10-x^2\right)^{3/2}}\,dx$

18. $\displaystyle\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx$

19. $\displaystyle\int \tan^{-1}x.\ \ell n\,(1+x^2)\,dx.$

20. $\displaystyle\int \frac{a+b\sin x}{(b+a\sin x)^2}\,dx$

21. $\displaystyle\int \frac{dx}{x^4\left(x^3+1\right)^2}$

22. $\displaystyle\int \frac{1+x\cos x}{x\left(1-x^2e^{2\sin x}\right)}\,dx$

23. $\displaystyle\int \frac{x\cos\alpha+1}{\left(x^2+2x\cos\alpha+1\right)^{3/2}}\,dx=\frac{f(x)}{\sqrt{g(x)}}+c$ then find $f(x)$ and $g(x)$

24. Evaluate $\displaystyle\int \frac{\ell n\,(1+\sin^2 x)}{\cos^2 x}\,dx.$

25. Integrate, $\displaystyle\int \frac{x^3+3x+2}{\left(x^2+1\right)^2(x+1)}\,dx.$ **[IIT - 1999, 7]**

26. For any natural number m, evaluate,

$$\int \left(x^{3m}+x^{2m}+x^m\right)\left(2x^{2m}+3x^m+6\right)^{1/m}dx,\ x>0.$$ **[IIT - 2002, 5]**

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ANSWER

EXERCISE–5

1. C **2.** A **3.** A **4.** A **5.** B **6.** A

7. B **8.** C **9.** B **10.** B **11.** C **12.** A

13. B **14.** C **15.** A **16.** B **17.** A **18.** D

19. AC *20.* *AB* *21.* *AD* **22.** ABCD **23.** BD

24. ACD

EXERCISE–6

1. $\ell n\left|1+2\tan\frac{x}{2}\right| + c$ **2.** $-\frac{1}{2\sqrt{3}}\ \ell n\left|\frac{x+\frac{1}{x}-\sqrt{3}}{x+\frac{1}{x}+\sqrt{x}}\right| + c$

3. $-\frac{1}{\sqrt{3}}\ \ell n\left|\left(t-\frac{1}{3}\right)+\sqrt{\left(t-\frac{1}{3}\right)^2+\frac{2}{9}}\right| + c$

where $t=\frac{1}{x+1}$

4. $\frac{1}{\sqrt{3}}\ \tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) - \frac{2}{\sqrt{3}}\ \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$

5. $2\ \ell n\left|\sin^2\phi-4\sin\phi+5\right| + 7\tan^{-1}(\sin\phi-2) + c$

6. $-\frac{1}{3}\ell n\,|1+\tan\theta| + \frac{1}{6}\ell n\,|\tan^2\theta-\tan\theta+1| + \frac{1}{\sqrt{3}}$

$\tan^{-1}\left(\frac{2\tan\theta-1}{\sqrt{3}}\right) + c$

7. $-(\sin x+\frac{\sin 2x}{2}) + c$

8. $2x - 3\arctan\left(\tan\frac{x}{2}+1\right) + c$

9. $x\cos\alpha + \sin\alpha\ \ell n\left\{\frac{\cos\frac{1}{2}(\alpha-x)}{\cos\frac{1}{2}(\alpha+x)}\right\} + c$

10. $\frac{-2}{\alpha-\beta}.\sqrt{\frac{x-\beta}{x-\alpha}} + c$

11. $\frac{\sqrt{x^2+2x-3}}{8(x+1)^2} + \frac{1}{16}.\cos^{-1}\left(\frac{2}{x+1}\right) + c$

12. $e^x\left(\frac{x+1}{x^2+1}\right) + c$

13. $c - \frac{1}{3}\ \tan x.(2+\tan^2 x).\sqrt{4-\cot^2 x}$

14. $\frac{(x^2+1)\sqrt{x^2+1}}{9x^3}.\left[2-3\ln\left(1+\frac{1}{x^2}\right)\right]$

15. $-\frac{b\sin x}{(a^2-b^2)(a+b\cos x)} + \frac{2a}{(a^2-b^2)^{3/2}}$

$\arctan\sqrt{\frac{a-b}{a+b}}.\tan\frac{x}{2} + c$

16. $\sin^{-1}\left(\frac{1}{2}\sec^2\frac{x}{2}\right) + c$

17. $\frac{2(7x-20)}{9\sqrt{7x-10-x^2}} + c$

18. $-\frac{\sqrt{2-x-x^2}}{x} + \frac{\sqrt{2}}{4}\ln\left(\frac{4-x+2\sqrt{2}\sqrt{2-x-x^2}}{x}\right)$

$-\sin^{-1}\left(\frac{2x+1}{3}\right) + c$

19. $x\tan^{-1}x.\ \ell n(1+x^2) + (\tan^{-1}x)^2 - 2x\tan^{-1}x$

$+\ \ell n(1+x^2) - \left(\ell n\sqrt{1+x^2}\right)^2 + c$

20. $-\frac{\cos x}{b+a\sin x} + c$

21. $\frac{2}{3}\ \ell n\left|\frac{x^3+1}{x^3}\right| - \frac{1}{3x^3} - \frac{1}{3(x^3+1)} + c$

22. $\ln(x\,e^{\sin x}) - \frac{1}{2}\ln(1-x^2e^{2\sin x}) + c$

23. $x;\ x^2 + 2x\cos\alpha + 1$

24. $\tan x\ln(1+\sin^2 x) - 2x + \sqrt{2}\tan^{-1}(\sqrt{2}.\tan x) + c.$

25. $\frac{3}{2}\ \tan^{-1}x - \frac{1}{2}\ \ell n(1+x) + \frac{1}{4}\ \ell n(1+x^2) + \frac{x}{1+x^2} + c$

26. $\frac{z^{\frac{m+1}{m}}}{6(m+1)}$ + c, where $z = 2x^{3m} + 3x^{2m} + 6x^m$

EXERCISE–7

Part : (A) Only one correct option

1. If f(x) is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non-zero x, then $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)\, dx$ equals

(A) $\sin\theta + \operatorname{cosec}\theta$ (B) $\sin^2\theta$ (C) $\operatorname{cosec}^2\theta$ (D) none of these

2. The value of the integral $\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1}$, where $0 < \alpha < \frac{\pi}{2}$, is equal to

(A) $\sin\alpha$ (B) $\alpha\sin\alpha$ (C) $\frac{\alpha}{2\sin\alpha}$ (D) $\frac{\alpha}{2}\sin\alpha$

3. If $\int_0^{100} f(x)\, dx = a$, then $\sum_{r=1}^{100}\left(\int_0^1 f(r-1+x)\,dx\right) =$

(A) 100 a (B) a (C) 0 (D) 10 a

4. If f(x) is an odd function defined on $\left[-\frac{T}{2}, \frac{T}{2}\right]$ and has period T, then $\phi(x) = \int_a^x f(t)\, dt$ is

(A) a periodic function with period $\frac{T}{2}$ (B) a periodic function with period T

(C) not a periodic function (D) a periodic function with period $\frac{T}{4}$

5. If $f(\pi) = 2$ and $\int_0^{\pi}(f(x) + f''(x))\sin x\, dx = 5$ then f(0) is equal to, (it is given that f(x) is continuous in [0, π])

(A) 7 (B) 3 (C) 5 (D) 1

6. If f(0) = 1, f(2) = 3, f′(2) = 5 and f′(0) is finite, then $\int_0^1 x \,.\, f''(2x)\, dx$ is equal to

(A) zero (B) 1 (C) 2 (D) none of these

7. $\lim_{n\to\infty}\left(\sin\frac{\pi}{2n}\,.\,\sin\frac{2\pi}{2n}\,.\,\sin\frac{3\pi}{2n}.......\sin\frac{(n-1)\pi}{n}\right)^{1/n}$ is equal to

(A) $\frac{\pi}{2}$ (B) $e^{4/\pi}$ (C) $e^{2/\pi}$ (D) none of these

8. f(x) = Minimum {tanx, cot x} $\forall\, x \in \left(0, \frac{\pi}{2}\right)$. Then $\int_0^{\pi/3} f(x)\, dx$ is equal to

(A) $\ell n\left(\frac{\sqrt{3}}{2}\right)$ (B) $\ell n\left(\sqrt{\frac{3}{2}}\right)$ (C) $\ell n(\sqrt{2})$ (D) $\ell n(\sqrt{3})$

9. If $A = \int_0^{\pi}\frac{\cos x}{(x+2)^2}\, dx$, then $\int_0^{\frac{\pi}{2}}\frac{\sin 2x}{x+1}\, dx$ is equal to

(A) $\frac{1}{2} + \frac{1}{\pi+2} - A$ (B) $\frac{1}{\pi+2} - A$ (C) $1 + \frac{1}{\pi+2} - A$ (D) $A - \frac{1}{2} - \frac{1}{\pi+2}$

10. $\int_{-\pi/2}^{\pi/2}\frac{|x|\, dx}{8\cos^2 2x + 1}$ has the value

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{24}$ (D) none of these

11. $\underset{n\to\infty}{\text{Lt}}\sum_{r=2n+1}^{3n}\frac{n}{r^2 - n^2}$ is equal to

(A) $\log\sqrt{\frac{2}{3}}$ (B) $\log\sqrt{\frac{3}{2}}$ (C) $\log\frac{2}{3}$ (D) $\log\frac{3}{2}$

12. If $\int_a^y \cos t^2 dt = \int_a^{x^2} \frac{\sin t}{t} dt$, then the value of $\frac{dy}{dx}$ is

(A) $\frac{2\sin^2 x}{x\cos^2 y}$ (B) $\frac{2\sin x^2}{x\cos y^2}$ (C) $\frac{2\sin x^2}{x\left(1-2\sin\frac{y^2}{2}\right)}$ (D) none of these

13. If $f(x) = \begin{cases} 0 , & \text{where } x = \frac{n}{n+1}, n = 1, 2, 3..... \\ 1 , & \text{else where} \end{cases}$, then the value of $\int_0^2 f(x)\,dx$

(A) 1 (B) 0 (C) 2 (D) ∞

14. $\int_0^1 \frac{x\,dx}{(1-x)^{3/4}} =$

(A) $\frac{15}{16}$ (B) $-\frac{16}{5}$ (C) $-\frac{3}{16}$ (D) none

15. Let $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ and $I_2 = \int_1^2 \frac{dx}{x}$, then

(A) $I_1 > I_2$ (B) $I_2 > I_1$ (C) $I_1 = I_2$ (D) $I_1 > 2I_2$

16. The value of $\int_0^{[x]} (x-[x])\,dx$ is

(A) $\frac{1}{2}[x]$ (B) $2[x]$ (C) $\frac{1}{2[x]}$ (D) none of these

17. The value of the integeral $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$ is equal to

(A) π (B) 2π (C) 4π (D) none of these

18. The value of $\int_0^{\pi/2} \log|\tan x+\cot x|\,dx$ is

(A) $\pi\log 2$ (B) $-\pi\log 2$ (C) $\frac{\pi}{2}\log 2$ (D) $-\frac{\pi}{2}\log 2$

19. If $\int_0^1 e^{x^2}(x-\alpha)\,dx = 0$, then

(A) $1 < \alpha < 2$ (B) $\alpha < 0$ (C) $0 < \alpha < 1$ (D) $\alpha = 0$

20. Suppose for every integer n, $\int_n^{n+1} f(x)dx = n^2$. The value of $\int_{-2}^{4} f(x)dx$ is :

(A) 16 (B) 14 (C) 19 (D) 21

21. Let $A = \int_0^1 \frac{e^t\,dt}{1+t}$ dt then $\int_{a-1}^{a} \frac{e^{-t}}{t-a-1}$ dt has the value :

(A) Ae^{-a} (B) $-Ae^{-a}$ (C) $-ae^{-a}$ (D) Ae^{a}

22. $\int_{5/2}^{5} \frac{\sqrt{(25-x^2)^3}}{x^4}$ dx equals to :

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) none

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23. The function $f(x) = \int_0^x \frac{dt}{t}$ satisfies **[IIT - 1996]**

(A) $f(x + y) = f(x) + f(y)$ (B) $f\left(\frac{x}{y}\right) = f(x) + f(y)$ (C) $f(xy) = f(x) + f(y)$ (D) none of these

24. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, $a > 0$ is

(A) π (B) $a\pi$ (C) $\pi/2$ (D) 2π

25. The integral $\int_{-1/2}^{1/2} \left([x] + \ell n\left(\frac{1+x}{1-x}\right)\right) dx$ equals:

(A) $-1/2$ (B) 0 (C) 1 (D) $2 \ln (1/2)$

26. If $I(m, n) = \int_0^1 t^m (1+t)^n \, dt$, then the expression of $I(m, n)$ in terms of $I(m + 1, n - 1)$ is **[IIT - 2003]**

(A) $\frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$ (B) $\frac{n}{m+1} I(m+1, n-1)$

(C) $\frac{2^n}{m-1} - \frac{n}{m+1} I(m+1, n-1)$ (D) $\frac{n}{m+1} I(m+1, n-1)$

27. If $\int_{\sin x}^{1} t^2 (f(t))\, dt = (1 - \sin x)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is **[IIT - 2005]**

(A) 1/3 (B) $1/\sqrt{3}$ (C) 3 (D) $\sqrt{3}$

28. $\int_{-2}^{0} \{x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\}\, dx$ is equal to **[IIT - 2005]**

(A) -4 (B) 0 (C) 4 (D) 6

Part : (B) May have more than one options correct

29. The value of integral $\int_0^{\pi} x f(\sin x)\, dx$ is

(A) $\pi \int_0^{\pi} f(\sin x)\, dx$ (B) $\pi \int_0^{\pi/2} f(\sin x)\, dx$ (C) 0 (D) none of these

30. If f(x) is integrable over [1, 2], then $\int_1^2 f(x)\, dx$ is equal to

(A) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (B) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

(C) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (D) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

31. If $f(x) = \int_0^x (\cos^4 t + \sin^4 t)\, dt$, $f(x + \pi)$ will be equal to

(A) $f(x) + f(\pi)$ (B) $f(x) + 2 f(\pi)$ (C) $f(x) + f\left(\frac{\pi}{2}\right)$ (D) $f(x) + 2f\left(\frac{\pi}{2}\right)$

32. The value of $\int_0^1 \frac{2x^2+3x+3}{(x+1)\left(x^2+2x+2\right)} dx$ is:

(A) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1} 2$ (B) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1}\frac{1}{3}$ (C) $2\ln 2 - \cot^{-1} 3$ (D) $-\frac{\pi}{4} + \ln 4 + \cot^{-1} 2$

33. Given f is an odd function defined everywhere, periodic with period 2 and integrable on every interval. Let $g(x) = \int_0^x f(t)\,dt$. Then:
(A) g(2n) = 0 for every integer n
(B) g(x) is an even function
(C) g(x) and f(x) have the same period
(D) none

34. If $I = \int_0^{\pi/2} \frac{dx}{\sqrt{1+\sin^3 x}}$, then
(A) $0 < I < 1$
(B) $I > \frac{\pi}{\sqrt{2}}$
(C) $I < \sqrt{2}\pi$
(D) $I > 2\pi$

35. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$; $n \in N$, then which of the following statements hold good?
(A) $2n\, I_{n+1} = 2^{-n} + (2n-1)\, I_n$
(B) $I_2 = \frac{\pi}{8} + \frac{1}{4}$
(C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$
(D) $I_3 = \frac{\pi}{16} - \frac{5}{48}$

EXERCISE–8

1. $\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n+1)x\, dx,\ n \in I$

2. If f, g, h be continuous functions on [0, a] such that f (a – x) = f (x), g (a – x) = – g (x) and 3 h (x) – 4 h (a – x) = 5, then prove that, $\int_0^a f(x)\, g(x)\, h(x) = 0$.

3. Assuming $\int_0^{\pi} \log \sin x\, dx = -\pi \log 2$, show that,
$$\int_0^{\pi} \theta^3 \log \sin\theta\, d\theta = \frac{3\pi}{2} \int_0^{\pi} \theta^2 \log\left(\sqrt{2}\sin\theta\right) d\theta.$$

4. Show that $\int_0^{\infty} f\left(\frac{a}{x}+\frac{x}{a}\right).\frac{\ln x}{x}dx = \ln a . \int_0^{\infty} f\left(\frac{a}{x}+\frac{x}{a}\right).\frac{dx}{x}$

5. Prove that $\int_0^x \left(\int_0^u f(t)\,dt\right) du = \int_0^x f(u).(x-u)\,du$.

6. Prove that $\int_0^{\infty} \frac{dx}{1+x^n} = \int_0^1 \frac{dx}{(1-x^n)^{1/n}}$ (n > 1)

7. Prove that $\lim_{n\to\infty} \frac{1}{n}\left[\cos^{2p}\frac{\pi}{2n} + \cos^{2p}\frac{2\pi}{2n} + \cos^{2p}\frac{3\pi}{2n} + ... + \cos^{2p}\frac{\pi}{2}\right] = \prod_{r=1}^{p} \frac{p+r}{4r}$

8. $\int_0^{\pi} \frac{x\, dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$

9. Evaluate $\int_0^1 |x-t| . \cos \pi t\, dt$ where 'x' is any real number

10. $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{\cos^{-1}\left(\frac{2x}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x+1}\, dx$

11. Evaluate, $I = \int_0^1 2\sin(pt)\sin(qt)\,dt$, if:

(i) p & q are different roots of the equation, tan x = x.

(ii) p & q are equal and either is root of the equation tan x = x.

12. Prove that $\int_0^x \frac{\sin x}{x+1}\, dx \ge 0$ for $x \ge 0$.

13. Let f(x) be a continuous functions $\forall\ x \in R$, except at x = 0 such that $\int_0^a f(x)dx$, $a \in R^+$ exists. If $g(x) = \int_x^a \frac{f(t)}{t}\, dt$, prove that $\int_0^a g(x)\, dx = \int_0^a f(x)\, dx$

14. If $f(x) = \frac{\sin x}{x}$ $\forall\ x \in (0, \pi]$, prove that, $\frac{\pi}{2}\int_0^{\pi/2} f(x)\, f\left(\frac{\pi}{2} - x\right) dx = \int_0^{\pi} f(x)\, dx$

15. Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$, x > 0. If $\int_1^4 \frac{2\, e^{\sin x^2}}{x}\, dx = F(k) - F(1)$ then one of the possible values of k is ______.

16. $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx.$ **[IIT - 1995, 5 + 2 + 2]**

17. Evaluate $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\, dx.$ **[IIT - 2005, 2]**

18. The value of $5050\,\frac{\int_0^1 (1-x^{50})^{100}\, dx}{\int_0^1 (1-x^{50})^{101}\, dx}$ is **[IIT - 2006, (6, 0)]**

ANSWER

EXERCISE–7

1. D **2.** C **3.** B **4.** B **5.** B **6.** C

7. B **8.** D **9.** A **10.** B **11.** B **12.** B

13. C **14.** D **15.** B **16.** A **17.** B **18.** A

19. C **20.** C **21.** B **22.** A **23.** C **24.** C

25. A **26.** A **27.** C **28.** C **29.** AB **30.** BC

31. AD **32.** AD **33.** ABC **34.** BC **35.** AB

EXERCISE–8

1. 0 **8.** $\frac{\pi^2\left(a^2+b^2\right)}{4a^3b^3}$

9. $-\frac{2}{\pi^2}\cos \pi x$ for $0 < x < 1$;

$\frac{2}{\pi^2}$ for $x \ge 1$ & $-\frac{2}{\pi^2}$ for $x \le 0$

10. $\frac{\pi}{2\sqrt{3}}$ **11.** (i) 0 (ii) $\frac{p^2}{1+p^2}$

15. 16 **16.** $\frac{\pi}{4}\ \ell n\left(2+\sqrt{3}\right) + \frac{\pi^2}{12} - \frac{\pi}{\sqrt{3}}$

17. $\frac{24}{5}\left[e\cos\left(\frac{1}{2}\right) + \frac{1}{2}e\sin\left(\frac{1}{2}\right) - 1\right]$ **18.** 5051